**Supplementary materials for**

**CHAPTER 2**

**of**

***The Biology and Conservation of Animal Populations***

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**A. SUMMARY POINTS**

A central goal of population biology is to understand how and why populations fluctuate in abundance. This understanding includes three abilities:

* describe past fluctuations (or dynamics) with useful accuracy and precision,
* evaluate ecological mechanisms that may have given rise to those dynamics, and
* make useful predictions about future dynamics.

Proportional growth: Population dynamics are importantly described as proportional (or percent) changes in abundance from one time period to the next. For example, a population that increases from 50 to 55 individuals has grown by 10%. A population that has grown from 500 to 550 has also grown by 10%.

Per capita growth rate: is a way of quantifying proportional growth. It is often represented by the symbol *r* and can be calculated using Equation (2.2):

*rt* = (*Nt*+1 – *Nt*)/*Nt*,

where *Nt* is abundance at time *t*. For many animal populations, *t* is measured annually. For example, if a population increases from 50 to 55 in a single year, then *rt* = 0.10/yr. *rt* always has units to indicate the time scale over which the growth is occurring.

Equation (2.2) can be algebraically re-arranged to predict next year’s abundance (*Nt*+1), given estimates for this year’s abundance (*Nt*) and this year’s per capita growth rate (*rt*):

*Nt*+1 = *Nt* + *Ntrt.*

This equation was introduced as Equation 2.1.

Density-independent growth: If a population’s per capita growth rate (*r*) does not tend to change as population size changes, then the population is said to have density-independent growth. A simple, informal way to check for density-independent growth is to create a graph with abundance (*Nt*) on the x-axis and *rt* on the y-axis. If there is no trend in the graph, then growth is density independent.

Exponential growth: Another way to describe population dynamics involves equation (2.4):

*Nt* = *Noert*.

This equation is associated with the label, *exponential growth*. (Equations [2.1] and [2.2] are associated with the label, *geometric growth*.) An important algebraic rearrangement of equation (2.4) is equation (2.5):

*r = ln(Nt*/*No)/t.*

If *r* is constant over time (or at least doesn’t exhibit much trend over time), then these equations are useful for describing a population’s dynamics.

Any population’s growth can be described using equations (2.1) and (2.2) or equations (2.4) and (2.5). Which pair of equations is best to use sometimes depends on what information is given to you about the population. But one shouldn’t mix and match those pairs of equations. For example, if you calculate *rt* using equation 2.2, then predict future abundances using equation 2.1. Otherwise, seek the advice of your instructor for other tips about when it’s best to use equations (2.1) and (2.2) or equations (2.4) and (2.5).

Log-transformed abundance:

* If a time series of abundance is growing (or declining) exponentially and if that time series is log-transformed, then the transformed time series will increase (or decrease) linearly.
* Many populations fluctuate over several orders of magnitude – ranging from very small to very large abundances. Imagine a time series of abundances for a population that spends some time fluctuating between 10 and 20 individuals, then grows to fluctuate for a time between 100 and 200 individuals. In that graph it will be difficult to appreciate the fluctuations between 10 and 20, because those values are all compressed at the bottom portion of the graph, like this:

But, if the time series that made that graph is log-transformed and then graphed, then more of fluctuations are easier to see. Like this

Both graphs are accurate depictions of the same population time series; but each graph can make it easier to see different aspects of a population’s dynamics. In particular, proportional fluctuations at lower levels of abundance are easier to discern in the lower graph than in the upper graph.

* Another connection between log-transformation and exponential growth may be seen in the reasoning given in the text that leads to Equation 2.7.

**B. DISCUSSION QUESTIONS**

1. In general terms, what ecological conditions are required for a population to exhibit density-independent population growth?
2. What is the most specific, real-world example you can think of (aside from those mentioned in Chapter 2) that would seem to favor density independent population growth?
3. How are birth rate and death rate related to per capita growth rate (r)?
4. Given that no population can exhibit density independent growth for too terribly long, what kinds of conservation issues are usefully understood through the lens of density independent growth?

**C. FURTHER READING**

Real-world examples best understood through the lens of exponential growth:

Leatherman, S. P., & Leatherman, S. B. (2024). Population Projections of Invasive Burmese Pythons in the Florida Everglades. *Journal of Coastal Research*, *40*(1), 223-227.

Van Bael, S., & Pruett-Jones, S. (1996). Exponential population growth of Monk Parakeets in the United States. *The Wilson Bulletin*, 584-588.

Blackwell, B. F., Stapanian, M. A., & Weseloh, D. C. (2002). Dynamics of the double-crested cormorant population on Lake Ontario. *Wildlife Society Bulletin*, 345-353.

Bowen, W. D., McMillan, J., & Mohn, R. (2003). Sustained exponential population growth of grey seals at Sable Island, Nova Scotia. *ICES Journal of marine science*, *60*(6), 1265-1274.

Gabriele, C. M., Neilson, J. L., Straley, J. M., Baker, C. S., Cedarleaf, J. A., & Saracco, J. F. (2017). Natural history, population dynamics, and habitat use of humpback whales over 30 years on an Alaska feeding ground. *Ecosphere*, *8*(1), e01641.

Some context associated with the example of decline in deep-sea fish:

Norse, E. A., Brooke, S., Cheung, W. W., Clark, M. R., Ekeland, I., Froese, R., ... & Watson, R. (2012). Sustainability of deep-sea fisheries. *Marine policy*, *36*(2), 307-320.

Ramirez-Llodra, E., Tyler, P. A., Baker, M. C., Bergstad, O. A., Clark, M. R., Escobar, E., ... & Van Dover, C. L. (2011). Man and the last great wilderness: human impact on the deep sea. *PLoS one*, *6*(8), e22588.

Victorero, L., Watling, L., Deng Palomares, M. L., & Nouvian, C. (2018). Out of sight, but within reach: A global history of bottom-trawled deep-sea fisheries from> 400 m depth. *Frontiers in Marine Science*, *5*, 98.

Conservation context associated with the whooping crane example:

Smith, EH. 2019. [Species Review: Whooping Crane](https://savingcranes.org/wp-content/uploads/2022/05/crane_conservation_strategy_whooping_crane.pdf). IUCN SSC Crane Specialist Group.

**D. PRACTICE PROBLEMS**

Use Excel, to build a table like Table 1 from the reading. But this time, make predictions for the elephants of Kruger National Park for the time period, 1929-1970. Your predictions can be made on the basis of this information: field-based estimates suggest that there were 105 elephants in 1929. Your predictions should also be based on these assumptions: that the recruitment rate was 0.10/year and that mortality rate was 0.015/year throughout the time period. In building this table, you’ll also confirm that *r* for the predicted dynamics is 0.085/year.

1. If a population has 125 individuals and a per capita growth rate of 0.05/year, how many organisms there will be next year? Use an equation from page 9 of Chapter 2. Round your answer to the nearest tenth of an individual. Your answer should include showing what equation you used.
2. The per capita growth rate (*r*) can be broken down into two more basic elements, i.e., the birth rate (*b*) and death rate (*d*) in this way: *r* = *b* – *d*.   
    Suppose that a population has a birth rate of 0.30 per year and a death rate of 0.45 per year. Further suppose the population has 83 individuals. Use the geometric population model to indicate how many organisms there will be next year. Round your answer to the nearest tenth of an individual.

In the Excel file that accompanies this assignment, you’ll find the data for estimated abundances of whooping cranes from 1953 to 2013. Use that data and Excel to calculate *rt* for each year, calculate the average value for the entire period, reproduce Figures 2.3a and 2.3b of Chapter 2, and make a graph that can assess whether *rt* has any tendency to increase or decrease over time.

Make the same calculations and graphs for the recovering wolf population depicted in Figs. 2.3c and 2.3d of Chapter 2. Again, you’ll find the data necessary to do so with the Excel file.

1. Below are data on the number of elephants across the continent of Africa over the past two centuries.

**Year # of elephants**

1800 27 million

1900 10 million

1920 5 million

1940 3 million

1970 2 million

1979 1.3 million

1989 600,000

1. 415,000

Use this data to answer these questions:

* 1. During which period of time has population decline, in terms of *r*, been the most severe? What was the annual rate of decline during that period?
  2. If elephants keep declining at the rate you identified for part A of this problem, when will there are only 100,000?
  3. If, beginning in 2016, elephant abundance began to increase at 0.025/year, by what year would one predict the population to recover to levels observed in the year 1900?

**Notice:** These data are fromwww.futureforelephants.org/en/information/the-crisis-in-africa

A moose population at low abundance and in good habitat can exhibit a per capita annual growth rate of 0.12/yr. If a population were growing at that rate, how much time would it take for abundance to double? To answer this question, start with the equation for exponential growth (*Nt* = *No*exp(*rt*)) and algebraically re-arrange that equation so that *t* is by itself on one side of the equation. Then apply the resulting equation to answer the question. Show the algebraic steps involved in manipulating the equation for exponential growth.

1. Use Excel to make a graph comparing the discrete geometric growth (eqn 2.1) and continuous exponential growth (eqn 2.4). The x-axis of the graph should run from time equal zero years to 20 years. The population size at time zero should be 100 individuals. And, the value of *r* should be set to 0.1/year. Display that graph in the Word document that contains answers to the other questions of this problem set. Describe the similarities and differences between the two growth trajectories.

**E. SOLUTIONS TO PRACTICE PROBLEMS**

1.

See associated Excel file for a solution.

2.

*Nt*+1 = *Nt* + *Ntrt* = 125 + (125 x 0.05) = 131.3

A population with 125 individuals that grows at an annual rate of 0.05/yr will have 131.3

individuals after a year.

3.

*Nt*+1 = *Nt* + *Nt* (*bt* – *dt*) = 83 + (83 x (0.30 – 0.45)) = 70.6

A population with 83 individuals, a birth rate of 0.30/year, and a death rate of 0.45/year

will have 70.6 individuals after a year.

4.

See associated Excel file for a solution.

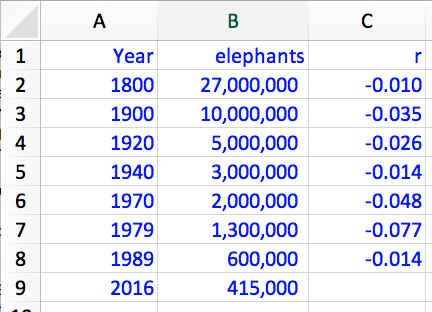
5.

See associated Excel file for a solution.

**6.**

**Answer to part A:** The most severe rate of decline occurred from 1979 to 1989. During that time the annual per capita population growth rate was –0.077/year.

The basis for this answer is the table below, which shows the per capita growth rate (*r*) for different periods of time.

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Cell C2 was calculated as: =LN(B3/B2)/(A3-A2), which is an application of equation 2.5 from the reading for this module.

**Answer to part B:** Use this equation, *t* = ln(*Nt*/*No*)/*r*. Replace *No* with 415,000, replace *Nt* with 100,000, and *r* with –0.077/year. Perform the calculation ln(100,000/415,000)/–0.077 = 18.5 years. On the basis of that calculation, one would predict that elephant abundance would drop to 100,000 between the years 2034 and 2035.

**Answer to part C:** Use the same equation as in part B. Replace *No* with 415,000. Replace *Nt* with 10,000,000. Replace *r* with 0.025/year. The result of that calculation is 127.3 years. This means one would predict abundance to be restored to 1900 levels by about the year 2143.

7.

*Start with:*

*Nt* = *No*exp(*rt*)

Divide both sides of that equation by *No*:

*Nt*/*No*= exp(*rt*)

Then take the natural log of both sides:

ln(*Nt*/*No*) = *rt*

Divide both sides by *r*:

ln(*Nt*/*No*)/*r* = *t,*

Because we’re interested in doubling time, replace *Nt*/*No* with 2

ln(2)/*r* = *doubling time*

ln(2)/0.12 = 5.8 years.

If a population has a growth rate of 0.12/yr, then its doubling time is 5.8 years. Wow, that’s fast.

**NOTE**: If a population is declining and you want to predict how quickly its abundance will be cut in half, then use this equation: halving time = ln(1/2)/*r*.

8.

